PATENT SPECIFICATION

DRAWINGS ATTACHED

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Int. Cl.:—F 02 k // B 29 f, j, F 23 r

COMPLETE SPECIFICATION

Filament Wound Pressure Vessel

We, AEROJET-GENERAL CORPORATION, a corporation duly organized and existing under the laws of the State of Ohio, United States of America, of 9190 East Flair Drive, El Monte, State of California, United States of America, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to a filament wound pressure vessel, e.g. a rocket motor casing. The word filament covers both mono-filament and multi-filament tensile members.

By designing the pressure vessel or rocket motor casing so that filaments are uniformly stressed in all locations, i.e. so that they are isotensoid, ultimate utilisation of the material is achieved. This in turn, means that the pressure vessel attains maximum reliability and, at the same time, it is of lesser weight than any other design.

This however, is not easy to achieve. The reason is that the shape of a filament wound pressure vessel is affected by its internal pressure, and any changes in the shape of the pressure vessel are likely to change the stress loadings on the filaments and disturb the uniformity of their loading. This is particularly true of the head or closure portion of the pressure vessel.

In addition, pressure vessels or rocket motor casings are commonly provided with openings in the apex of their head portions. Changes in the size of the openings in otherwise identical heads would affect the stress pattern in the filaments so that both the size of the head portions of the pressure vessel and the size of the openings in the head portions are important factors affecting the stress in the filament. Consequently a theoretical approach based on an idealized head shape without considering the apex opening may result in a wide variation between the theoretical strength of the vessel and its actual strength. What is needed, therefore, is a filament wound pressurised vessel having filament stresses which remain balanced and equal regardless of the internal pressure in the shell.

This invention consists in a pressure vessel formed from wound filaments comprising at least identical opposed head portions, each head portion being provided with a single apex opening and comprising a surface of revolution obtained by rotating a meridian curve about an axis of revolution lying in the plane of the curve, said meridian curve substantially defined by the expression

$$y = -\int \frac{x^3\,dx}{\sqrt{(1-x^2)(x^2-a_1)(x^2-a_2)}} + k$$

where y is measured along the axis of revolution

x is measured transverse to the axis of revolution

k is a constant of integration evaluated by the requirement that when $y=0$, x equals its maximum value of 1

$$a_1 = \frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}} - 1\right)$$

$$a_2 = -\frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}} + 1\right)$$

and x_o equals the radius of the apex opening in each of the head portions, and means associated with said filaments to form a pressure-tight seal over the surface area of said vessel, said filaments forming said opposed head portions and being oriented to lie along geodesic lines so that the stresses in the filaments are equal and remain equal independently of the pressurization of the vessel, whereby the stress pattern in the filament matrix is independent of the pressurization of the

vessel due to the deformed configuration of the vessel under pressurization being geometrically proportional to the undeformed configuration thereof.

The invention will be further described, by way of example, with reference to the accompanying drawings, of which:—

Figure 1 is a perspective view of a pressure vessel according to this invention;

Figure 2 is a perspective view of the pressure vessel of Figure 1, but in a preliminary stage of its construction;

Figure 3 is a graph showing a plurality of meridian curves each corresponding to an opening of a predetermined size in the head of the pressure vessel, the graph coordinates being normalized (i.e. at right angles to each other);

Figure 4 is a digram showing the geometry of the head shape; and

Figure 5 is a cross-sectional view of a rocket motor according to this invention.

Referring now to Figure 1 of the drawings, an isotensoid pressure vessel 10 comprises identical opposed head portions 12 and 14. In this particular embodiment there is also included an intermediate generally cylindrical portion 16 joined to the bases or equators 17 of the head portions. The head portions 12 and 14 are surfaces of revolution defined by the rotation of the meridian curve 18 around an axis of revolution 20 (see Figure 3). In addition, the head portions 12 and 14 are provided with identical openings 22 disposed at their apices and concentric with the axis. As seen in Figure 3, the selection of the radius X_o of the opening 22 determines the meridian curve 18 and hence the contour of the head.

In order to provide an isotensoid filament wound pressure vessel, wherein the filaments remain isotensoid regardless of the internal pressure in the shell, the contour of the head and the filament wrap pattern must satisfy certain conditions. In particular, the deformed configuration of the vessel when under pressure must be geometrically proportional to its undeformed configuration. This requires a consideration of the stress distribution in the heads of pressurised vessels.

In general, for surfaces of revolution, the stresses in the head of a pressurised vessel can be resolved into two principal directions. These are the meridional direction and hoop direction, indicated respectively generally by the arrows 24 and 26 (see Figure 4). From theoretical considerations, when the pressure loading in the head of a filament wound pressure vessel is in equilibrium, the magnitude of the force in the meridional direction, as shown by the arrows 24 in Figure 4 and indicated symbolically by the notation N_ϕ is

$$N_\phi = \frac{Pr_2}{2} \qquad (1)$$

In addition, the magnitude of the force in the hoop direction, as shown by arrows 26 in Figure 4 and indicated symbolically by the notation N_θ is

$$N_\theta = \frac{Pr_2}{2}\, 2 - \frac{r_2}{r_1} \qquad (2)$$

where P is the internal pressure in the shell,

r_1 is the meridional radius of curvature of the head at a point thereon, and

r_2 is the circumferential radius of curvature of the head at the same point.

In an isotensoid filament wound pressure vessel all filaments carry the same load. This load may be designated as L_f. If at a point on the head, the number of filaments contained in an area of unit length and thickness t_o is called m, the stress in the meridional direction

$$N_\phi = m\, L_f \cos^2 \alpha \qquad (3)$$

and the stress in the hoop direction

$$N_\theta = m\, L_f \sin^2 \alpha \qquad (4)$$

where α is the angle between the direction of the filaments 29 and the meridional direction on the head indicated by arrow 27 (see Figure 4). The ratio of these stress resultants is

$$\frac{N_\theta}{N_\phi} = \tan^2 \alpha \qquad (5)$$

but from equations 1 and 2 this is equal to

$$\frac{(2-r_2)}{r_1} \qquad (6)$$

It can be shown from mathematical considerations that for a surface of revolution

$$r_1 = \frac{-a\,[1+(y'^2)]^{3/2}}{y''} \qquad (7)$$

and

$$r_2 = \frac{-ax\,[1+(y')^2]^{1/2}}{y'} \qquad (8)$$

where

a in the mathematical expression equals the radius of the head at the equator 17,

y is measured along the axis of revolution,

y' is the first derivative of y with respect to x,

y'' is the second derivative of y with respect to x, and

x is the radial distance of a point on the head from the axis of revolution.

From (5) and (6)

$$\tan^2 \alpha = 2 - \frac{r_2}{r_1} \qquad (9)$$

Substituting the values of x, y', and y'' for r_2 and r_1 as defined by (7) and (8), it is apparent that the $\tan^2 \alpha$ is a function of x, y' and y'', i.e.,

$$2 - \tan^2 \alpha = \frac{x\, y''}{y' [1 + (y')^2]} \qquad (10)$$

In order to provide an isotensoid pressure vessel wherein the filament stresses remain balanced independently of the internal pressure in the vessel, the filaments forming the surface of the head of the pressurised vessel must be wound along geodesic lines. This condition is stated mathematically by the expression

$$x \sin \alpha = \text{constant} \qquad (10a)$$

The constant can be evaluated because the heads are provided with identical apex openings of radius x_o and the filaments must be tangent to the opening 22. Consequently,

$$x \sin \alpha = x_o \qquad (11)$$

Since α has been shown to be a known function of x, y', and y'', the substitution of x_o and x for α provides a differential equation which can be solved. In other words from (10) to (11)

$$\frac{x\, y''}{y' [1 + (y')^2]} = \frac{2x^2 - 3x_o^2}{x^2 - x_o^2} \qquad (12)$$

Using standard mathematical procedures, differential equation (12) can be integrated and solved for y as a function of x.

This expression which defines the desired meridian curve is

$$y = -\int \frac{x^3\, dx}{\sqrt{(1 - x^2)(x^2 - a_1)(x^2 - a_2)}} + k$$

where

$$a_1 = \frac{1}{2}\left(\sqrt{1 + \frac{4x_o^2}{1 - x_o^2}} - 1\right) \qquad (13)$$

and

$$a_2 = -\frac{1}{2}\left(\sqrt{1 + \frac{4x_o^2}{1 - x_o^2}} + 1\right)$$

and

k is evaluated from the fact that in the heads when $y=0$, the value of $x=1$. Equation (13), defining the merdian curve, is an elliptic integral of the third kind which can be programmed readily on a computer. It is apparent from (13) that the meridian curve 18 varies in accordance with the size of the opening in the head.

If the pressure vessel includes an intermediate cylindrical or barrel shaped section 16, the angle α_o at the equator of the head equals the $\sin^{-1} x_o$ (see Figure 4). This angle is maintained constant over the length of the cylinder because all lines with a constant angle α_o are geodesics on a cylindrical surface. According to equation (1), the meridional force N_ϕ on a pressure vessel equals $\frac{Pr_2}{2}$ and since the radius "a" of the cylindrical portion 16 of the pressure vessel 10 is equal to the radius of the equator of the head, the meridional force or longitudinal loading on the cylinder equals

$$N_{\theta_o} = \frac{P\, a}{2} \qquad (14)$$

If m is the number of filaments on the equator which are conained in an area of unit length and width t_o, and the filament fibres carry a constant load Lf, the total meridional force or longitudinal loading N_{ϕ_o} on the cylindrical surface is from equation (3)

$$m\, L_f \cos^2 \alpha_o \qquad (15)$$

Since a pressurised cylinder is stressed in the hoop direction as well as in the meridional direction, the existing filaments have to be supplemented by additional hoop windings. If the number of hoop windings within a unit length of the cylinder is called n, then the hoop load N_{θ_o} is, from equation (4),

$$N_{\theta_o} = P\, a = mL_f \sin^2 \alpha_o + nL_f \qquad (16)$$

where P is the internal pressure of the vessel and a is the radius of the cylindrical portion. From these expressions the ratio of the hoop filaments to filaments at an angle α_o can be found. As $\sin^2 \alpha_o + \cos^2 \alpha_o = 1$, it can be derived from equations (14), (15) and (16) that this ratio $\frac{n}{m}$ is

$$\frac{n}{m} = 3 \cos^2 \alpha_o - 1 \qquad (17)$$

An analysis of the meridian curve which defines the curvature of the head shape discloses the presence of a point of inflection at $x=x_o\sqrt{3/2}$. Consequently, the meridional curve is not applicable for smaller values of x than this. Therefore, in the region $x_o<x<x_o\sqrt{3/2}$ at the vicinity of the opening 22, additional reinforcement is required in the form of an insert to distribute the meridional load.

As illustrated in Figures 1 and 2, to fabricate the pressure vessel or rocket motor casing according to the principles of this invention, a mandrel 28, having opposed head portions comprising surfaces of revolution defined by the above described meridian curve, is first formed. The mandrel 28 is preferably made from a liquid soluble material such as salt. Then sections of uncured neoprene or other suitable materials are laid on the surface of the mandrel, except over apex circles on the heads of the mandrel which define the periphery of the apex openings in the heads of the completed vessel. The neoprene is then cured to form an intgral gas-tight liner 31 on the surface of the mandrel. Next, the mandrel is slowly rotated and the filaments are wound on it so they lie along geodesic paths. This continues until the thickness of the filaments wound over the entire surface of the mandrel is sufficient to withstand the expected internal pressure. Figure 2 shows that none of the windings pass over circular portions on the apex of the heads of the mandrel, which in the completed pressure vessel, will form the apex openings 22 in the heads. With this arrangement, when the vessel 10 is pressurised, its deformed shape will be geometrically proportional to its unpressurised shape. Consequently, the stress pattern in the filament matrix is uniform throughout.

If the pressure vessel is fabricated from resin-impregnated filaments such as resin-impregnated glass, after the filaments are wound over the surface of the mandrel to a sufficient thickness, the resin-impregnated filaments are cured in a manner well known in the art to cause the filaments to stick together and give rigidity to the pressure vessel. After this, the mandrel may be removed by dissolving it in a suitable liquid poured through the openings in the heads of the vessel.

It is noted that head contours designed according to the principles of this invention allow for the apex openings 22. Consequently, these openings do not cause a variance between the calculated strength of the pressure vessel and the actual strength.

As stated above, the principles of this invention can be applied to the formation of a rocket motor casing. In Figure 5, a rocket motor casing 29*a* comprises opposed head portions 30 and 32, designed in accordance with the principles of this invention. In addition, an intermediate generally cylindrical or barrel shaped portion 34 is provided. The head portions 30 and 32 are provided with the usual identical openings 36 and 38 respectively. A closure element or injection plate 40 may be mounted in the opening 36 in the head portion 30. Similarly, a nozzle portion 42 may be mounted in opening 38 in head portion 32. If the contours of the heads 30 and 32 are defined by the meridian curves, described above, and filaments composing the rocket motor housing are mounted along geodesic lines, the weight of the rocket motor casing can be minimized so the resulting rocket motor housing can have a burst strength only slightly greater than the strength required for the anticipated maximum. In addition, the geodesic windings remain equally stressed throughout severe pressure fluctuations in the rocket motor combustion chamber.

WHAT WE CLAIM IS:—

1. A pressure vessel formed from wound filaments comprising at least identical opposed head portions, each head portion being provided with a single apex opening and comprising a surface of revolution obtained by rotating a meridian curve about an axis of revolution lying in the plane of the curve, said meridian curve substantially defined by the expression

$$y=-\int\frac{x^3\,dx}{\sqrt{(1-x^2)(x^2-a_1)(x^2-a_2)}}+k$$

where y is measured along the axis of revolution

x is measured transverse to the axis of revolution

k is a constant of integration evaluated by the requirement that when $y=0$, x equals its maximum value of 1

$$a_1=\frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}}-1\right)$$

$$a_2=-\frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}}+1\right)$$

and x_o equals the radius of the apex opening in each of the head portions, and means associated with such filaments to form a pressure-tight seal over the surface area of said vessel, said filaments forming said opposed head portions and being oriented to lie along geodesic lines so that the stresses in the filaments are equal and remain equal independently of the pressurization of the vessel, whereby the stress pattern in the filament matrix is independent of the pressurization of the vessel due to the deformed configuration of the vessel

under pressurization being geometrically proportional to the undeformed configuration thereof.

2. A pressure vessel as claimed in claim 1 and comprising an intermediate portion generally circular in cross-section.

3. A pressure vessel as claimed in claim 2, wherein said intermediate portion is generally cylindrical.

4. A pressure vessel as claimed in any one of the preceding claims, wherein the filaments on said surfaces of the head portions are wound so they lie along geodesic lines on the surface at an angle α with respect to meridional lines wherein at any point on the surfaces of the head portions, α is defined as the angle whose sine is equal to the ratio of x_o to x.

5. A pressure vessel as claimed in claim 4 when read as appendant to claim 2, wherein said intermediate portion is generally cylindrical and wherein said filaments continue from the head portions over the generally cylindrical portion at an angle α_o with respect to the meridional lines thereon where α_o is the angle the filaments make with the meridional lines at the equator of each of the head portions and additional hoop wound filaments over the intermediate cylindrical portion to resist hoop stresses therein.

6. A method of fabricating an isotensoid pressure vessel comprising the steps of forming a mandrel having opposed identical head portions, each head portion comprising a surface of revolution obtained by rotating a meridian curve about an axis lying in the plane of the curve, said meridian curve defined by the expression

$$y = -\int \frac{x^3\,dx}{\sqrt{(1-x^2)(x^2-a_1)(x^2-a_2)}} + k$$

where

y is measured along the axis of revolution

x is measured transverse to the axis of revolution

k is a constant of integration evaluated by the requirement that when $y=o$, x equals its maximum value of 1

$$a_1 = \frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}} - 1\right)$$

$$a_2 = -\frac{1}{2}\left(\sqrt{1+\frac{4x_o^2}{1-x_o^2}} + 1\right)$$

and x_o equals the radius of a circle on the apex of each of the head portions which defines the respective periphery of an intended opening in each of the head portions of the vessel, wrapping a gas-tight liner on the surface of the mandrel except over the apex circles on the apices of the head portions, winding resin impregnated filaments over the liner on the head portions so they lie along geodesic lines and are tangent to the circles on the apices of the head portions, curing the resin impregnated filaments to give rigidity to the casing formed thereby, and removing the mandrel to complete the vessel.

7. A pressure vessel fabricated by the method of claim 6.

8. A pressure vessel as claimed in any one of claims 1 to 5 and 7, wherein the apex opening in one head portion is closed off by a closure member and a nozzle portion is fitted in the apex opening in the opposite head portion to form a rocket motor casing.

9. A pressure vessel, substantially as herein described, with treference to, and as shown in, Figures 1 to 4 of the accompanying drawings.

10. A rocket motor casing, substantially as herein described with reference to, and as shown in, the accompanying drawings.

11. A method of fabricating a pressure vessel, substantially as herein described with reference to the accompanying drawings.

MARKS & CLERK,
Chartered Patent Agents,
Agents for the Applicants.

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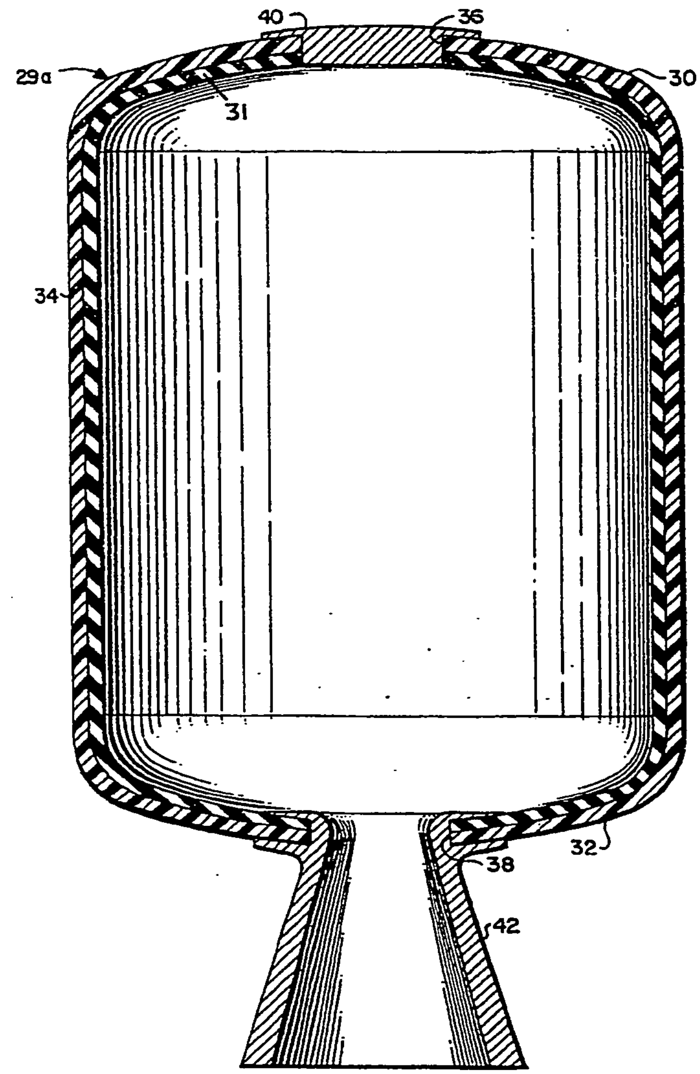

Fig-5

1064590 COMPLETE SPECIFICATION

2 SHEETS *This drawing is a reproduction of the Original on a reduced scale*

Sheet 1

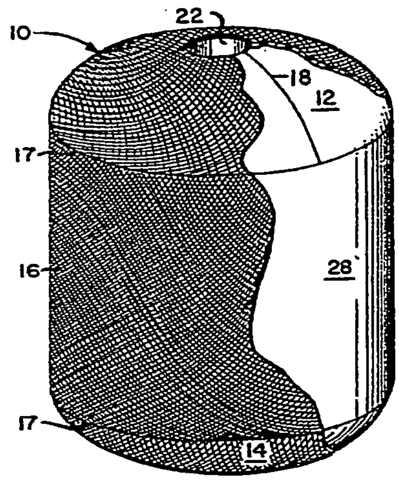

Fig-1

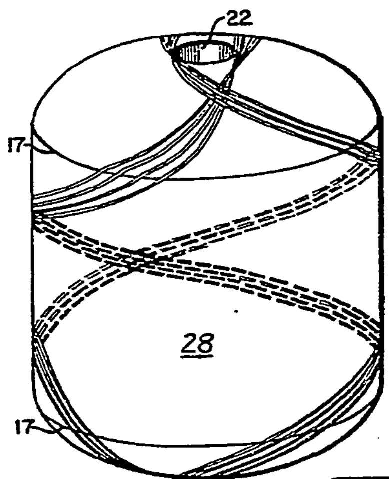

Fig-2

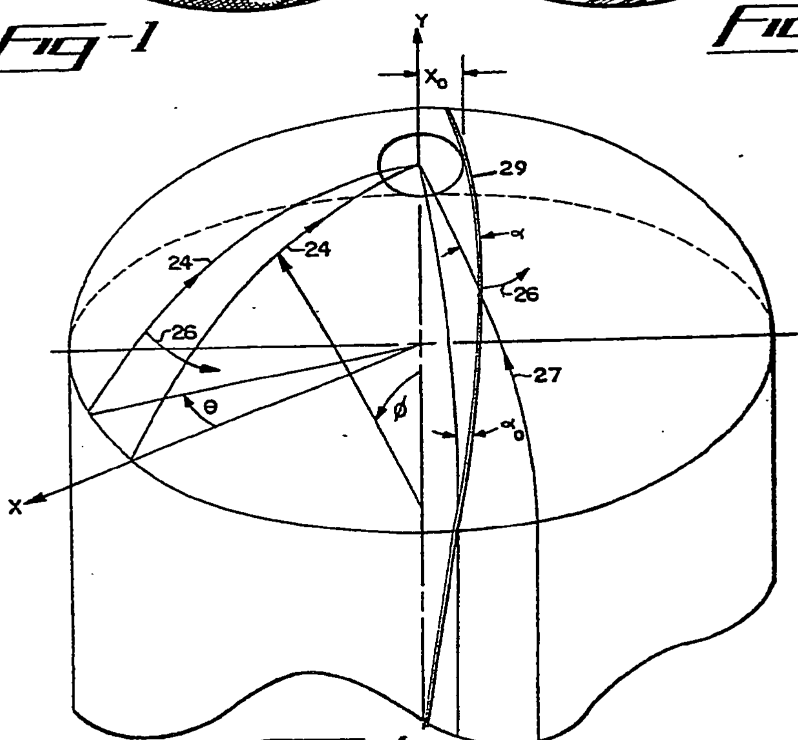

Fig-4

1064590 COMPLETE SPECIFICATION

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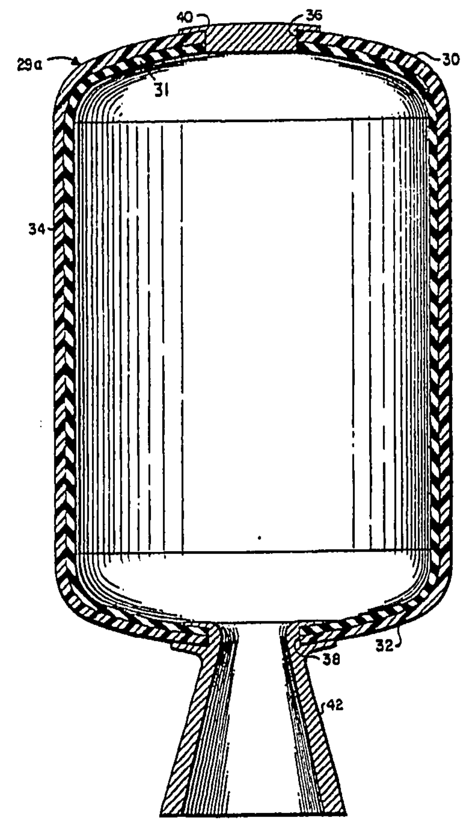


Fig-5

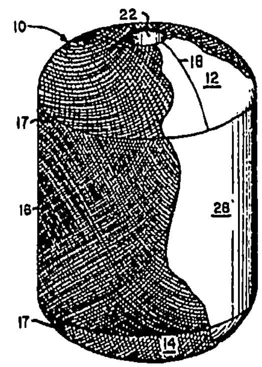


Fig-1

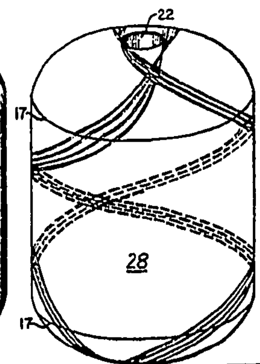


Fig-2

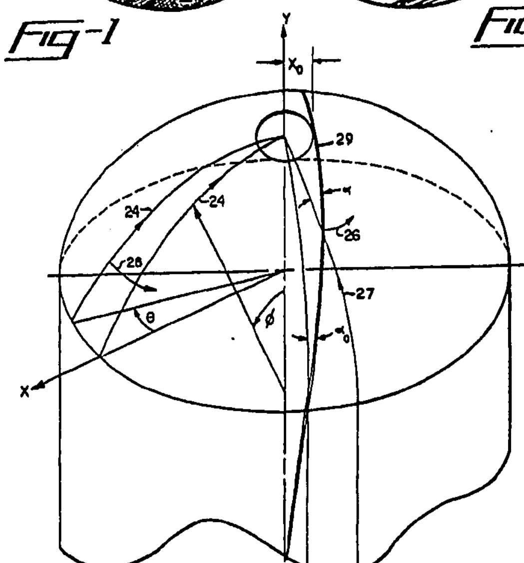
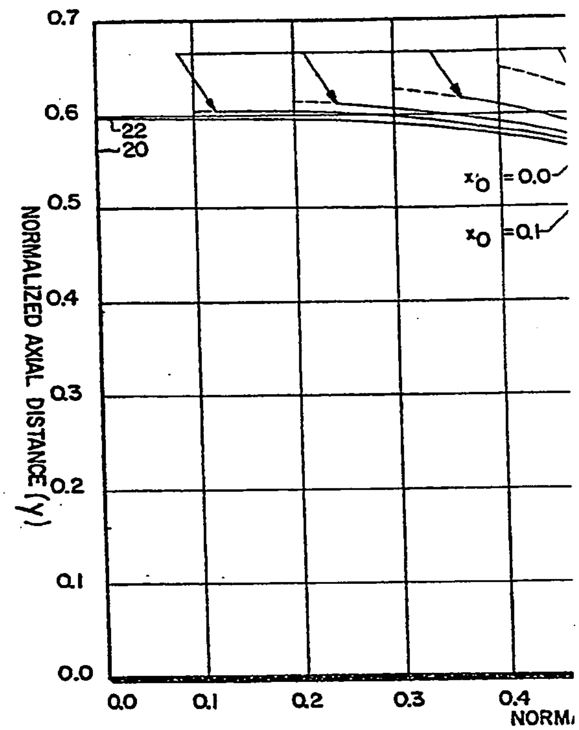


Fig-4



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
22
20
x_0 =0.0
x_0 =0.1
NORMALIZED AXIAL DISTANCE (y)
0.0
0.1
0.2
0.3
0.4
NORM

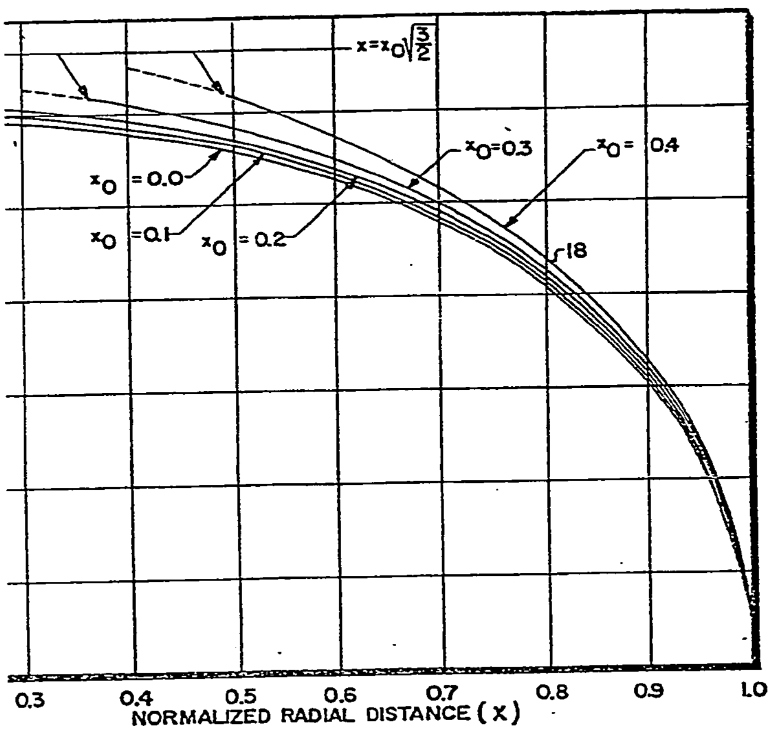

x=x0√(3/2)
x0 = 0.0
x0 = 0.1
x0 = 0.2
x0=0.3
x0= 0.4
18
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
NORMALIZED RADIAL DISTANCE (X)

Fig-3

1064590 COMPLETE SPECIFICATION

2 SHEETS *This drawing is a reproduction of the Original on a reduced scale*

Sheet 2

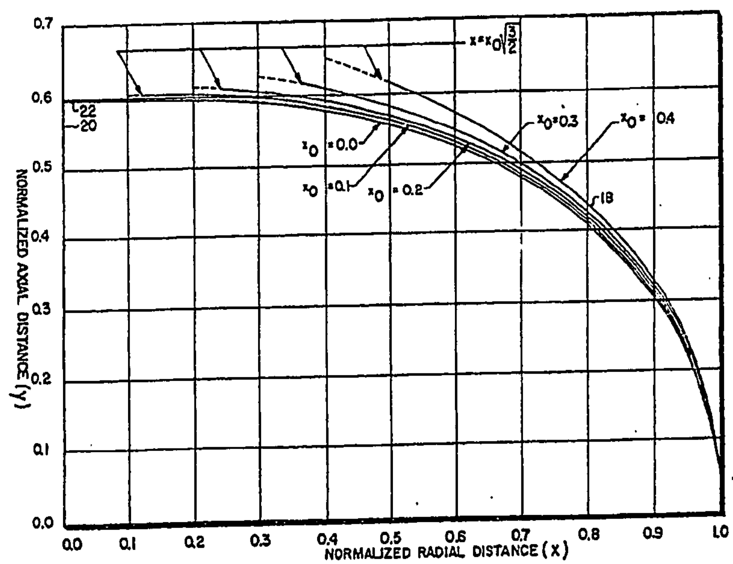


Fig-3